USE OF ANALYTIC GEOMETRY FOR TASK SOLUTION ON MANEUVERING BOARD

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ABSTRACT
Nowadays, **maneuvering board** is still taught in maritime institutions. This aid to navigation provides a **graphical solution** for determining true-motion and relative-motion factors of vessels.
The project of the author is focused on the method of navigation task solution at maritime academic institutions. Besides using graphical solution as an initial step of training, **analytic geometry** can be used as a secondary or control method.
In navigation, a navigational polar coordinate system is used to determine an object position with a bearing (BRG, in degrees) and a range (RNG, in nautical miles). A position can be converted into a point in a Cartesian coordinate system, i.e. with two axes x and y.
Analytic geometry is used to make up some formulas to determine those movement factors of other targets, e.g. CPA (closest point of approach, in nautical miles), $TCPA$ (time to CPA, in minutes), heading/course (in degrees), speed (in knots), etc.

1. THE BRIEF HISTORY

The table below shows the brief history of the radar and its use:

Table 1. The brief history

Time	Event
In the 1880s and the 1890s	There were first experiments of many famous physicists from all over the world, using radio wave to detect objects.
World War I and World War II	There were the first versions of modern radar developed.
*	
In the 1960s	The first versions of ARPA (*Automatic radar plotting aid*) were developed and applied in shipping industry.

The **gap period** in the table, which is marked by an asterisk, is a big question to the author. In that period, **maneuvering board** was used to rapidly determine motion factors.

2. CONVERTING BETWEEN NAVIGATIONAL POLAR AND CARTESIAN COORDINATES

In order to convert a position from one coordinate system to another, some trigonometric functions are applied.

For the purpose of this project, the navigational polar and Cartesian coordinate systems are mentioned.

The concentric circles with equal intervals of distance can be used to rapidly determine the range for the purpose of navigation.

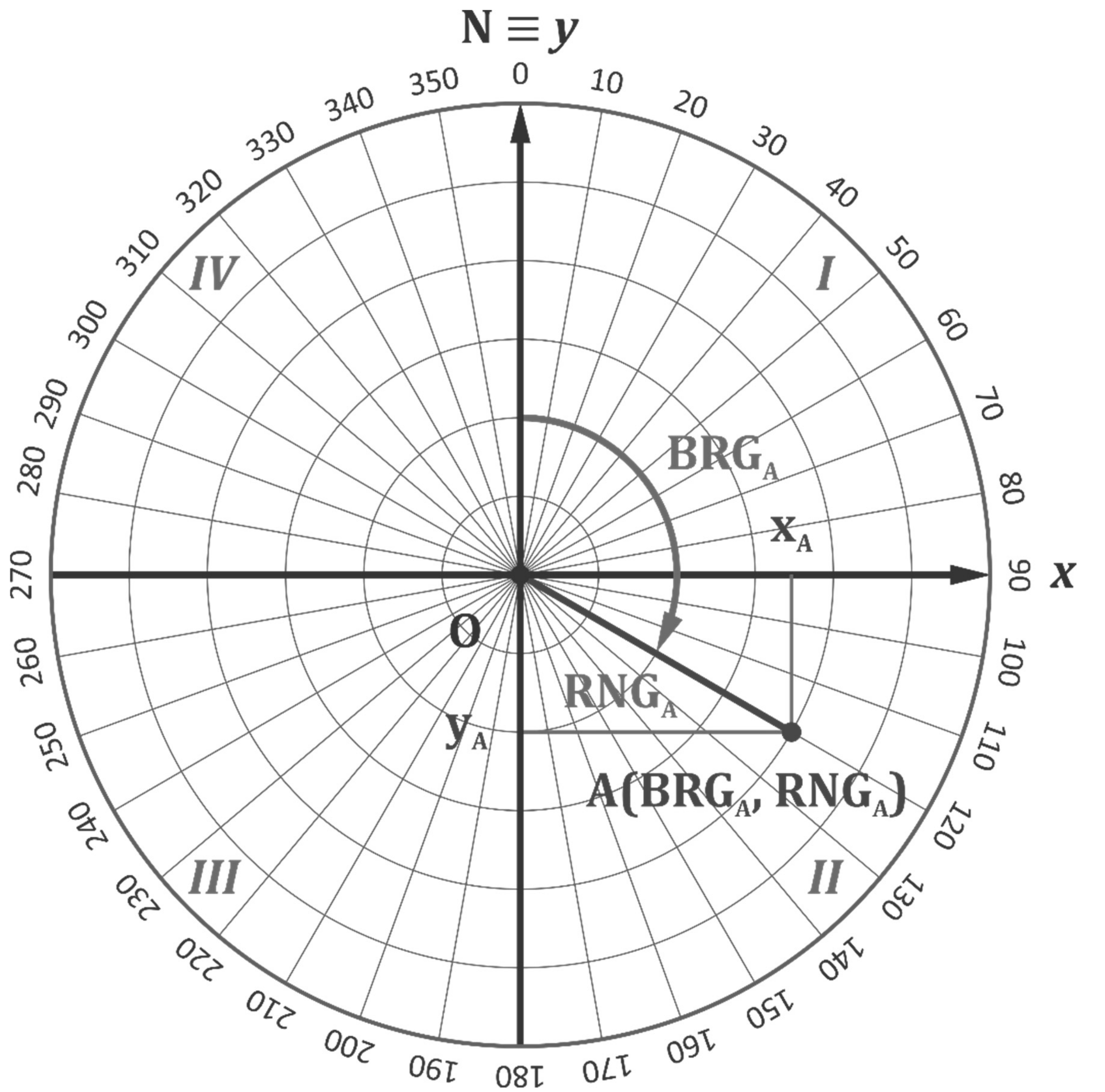

Figure 1. Converting between navigational polar and Cartesian coordinates

The vertical axis Oy and the north axis ON are coincident, which point to a direction of $BRG = 0°$. Consequently, the horizontal axis Ox points to a direction of $BRG = 90°$.
As to the conversion formulas mentioned in this case, there is a change in the applied trigonometric functions sine and cosine, because of the main differences from the ordinary polar coordinate system (i.e. the north axis position and the direction of the bearing angle).
The table below shows the main basic formulas used to convert between navigational polar and Cartesian coordinates.

Table 2. Converting formulas

Navigational polar coordinate	**Cartesian coordinate**
$RNG = r = \sqrt{x^2 + y^2}$ $BRG = \arctan\left(\frac{x}{y}\right) = \arcsin\left(\frac{x}{RNG}\right) = \arccos\left(\frac{y}{RNG}\right)$ *(with defined conditions)*	$x = RNG \cdot \sin BRG$ $y = RNG \cdot \cos BRG$

"*The defined conditions*" mean the definition for quadrant determination, which depends on the signs of the coordinates x and y:

Tables 3. Quadrants

Quadrant	***I***	***II***	***III***	***IV***
x	$x \geq 0$	$x \geq 0$	$x \leq 0$	$x \leq 0$
y	$y \geq 0$	$y < 0$	$y < 0$	$y \geq 0$
$\boldsymbol{BRG}$	$\arcsin z$ $\arccos z$ $\arctan z$	$\arcsin z + 90°$ $\arccos z$ $\arctan z + 180°$	$180° - \arcsin z$ $360° - \arccos z$ $\arctan z + 180°$	$\arcsin z + 360°$ $360° - \arccos z$ $\arctan z + 360°$

where: z – respective trigonometric functions of BRG

3. MANUAL GRAPHICAL SOLUTION [1]

The figure and the table show an example of manual graphical solution on manoeuvring board and its description.
For the purpose of this project, the central point $O(0, 0)$ represents the own vessel, and the points with alphabetical names, e.g. $A(BRG_A, RNG_A)$, show the other objects or vessels (in general, targets).

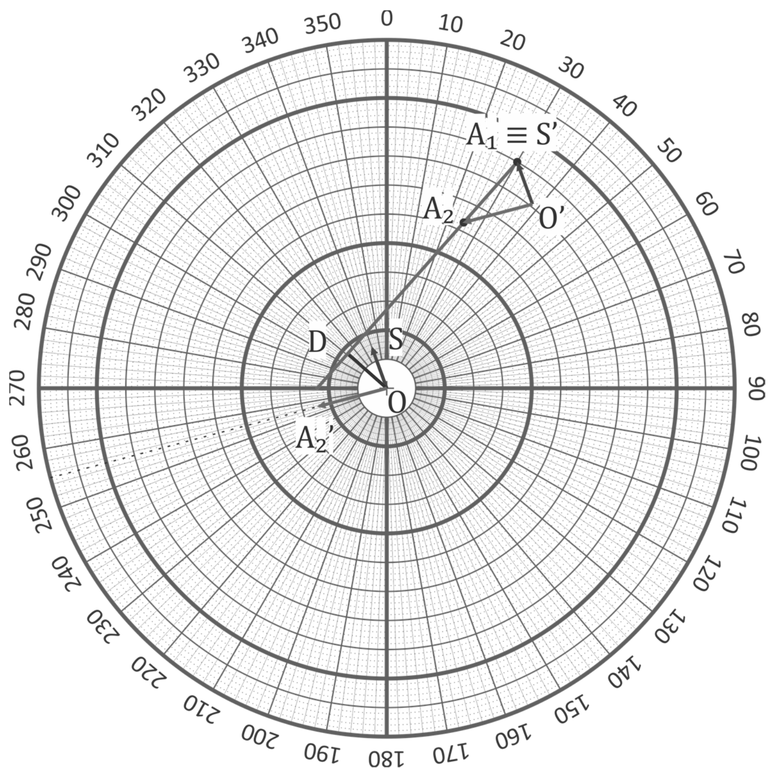

Figure 2. An example of manual graphical solution on manoeuvring board

Table 4. Manual graphical solution description

Step	Description	Remark
1	**Situation**	
	The own ship data (heading/ course and speed): $HDG = CSE = 340°$ $SPD = 15\ (kn)$ **The target *A* data:** At moment t_1 $\begin{cases} t_1 = 00:00:00 \\ A_1(030°, 9.0') \end{cases}$ At moment t_2 $\begin{cases} t_2 = 00:06:00 \\ A_2(025°, 6.3') \end{cases}$ Interval of time between moments $\Delta t = 6\ (min)$ **Required movement factors:** $A(CPA, TCPA, CSE_A, SPD_A) = ?$	

2	Plotting the own ship vector	
	$Scale = \frac{\Delta t}{60} = \frac{6}{60} = 1:10$ $\overrightarrow{OS} = (360°, 15\ kn) \longrightarrow \overrightarrow{OS} = (360°, 1.5\ nm)$	
3	Plotting the target relative vector	
	$A_1(030°, 9.0')$ $A_2(025°, 6.3')$ The target relative vector is $\overrightarrow{A_1A_2}$ vector.	
4	Determining the target CPA	
	$\boldsymbol{CPA = OD \approx 1.8\ (nm)}$	
5	Determining the target $TCPA$	
	$\boldsymbol{TCPA = \frac{A_2D}{A_1A_2/\Delta t} = \frac{6}{2.7/6} \approx 13.3\ (min)}$	
6	Determining the target true vector	
	$\boldsymbol{CSE_A \approx 252.5°}$ $\boldsymbol{SPD_A \approx 25\ kn}$	

4. APPLICATION OF ANALYTIC GEOMETRY ON MANEUVERING BOARD

The initial analytic geometry formulas for determining movement factors are converted into final formulas, which depend only on input data, without any other intermediate factors.

Initial formulas:

$$CPA = OD = |\overrightarrow{OD}| = \sqrt{x_D{}^2 + y_D{}^2}$$

$$TCPA = \frac{A_2D}{A_1A_2} \cdot \Delta t$$

$$SPD_A = 10 \cdot \sqrt{x_2{}'^2 + y_2{}'^2}$$

$$CSE_A = \left(\widehat{Oy, \overrightarrow{OA_2'}}\right) = arctan\frac{x_2'}{y_2'} = arcsin\frac{x_2'}{V_A/_{10}} = arccos\frac{y_2'}{V_A/_{10}}$$

(with defined condition)

Final formulas:

$$\boldsymbol{CPA = \frac{|R_1 \cdot R_2 \cdot \sin(B_1 - B_2)|}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}$$

$$\boldsymbol{TCPA = \frac{\sqrt{{R_2}^2 - \frac{[R_1 \cdot R_2 \cdot \sin(B_1 - B_2)]^2}{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}}}{\sqrt{{R_1}^2 + {R_2}^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos(B_1 - B_2)}} \cdot \Delta t}$$

$$\boldsymbol{SPD_A = 10 \cdot \sqrt{\left(R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H\right)^2 + \left(R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H\right)^2}}$$

$$\boldsymbol{CSE_A = arctan\frac{R_2 \cdot \sin B_2 - R_1 \cdot \sin B_1 + \frac{V}{10} \cdot \sin H}{R_2 \cdot \cos B_2 - R_1 \cdot \cos B_1 + \frac{V}{10} \cdot \cos H}}$$

(with defined condition)

where: R – range, B – bearing, V – the own ship's speed, H – the own ship's heading

Table 5. The comparison

Value	**Manual graphical solution**	**Analytic geometry**
CPA	$1.8\ (nm)$	$1.780\ (nm)$
$TCPA$	$13.3\ (min)$	$13.049\ (min)$
CSE	$252.5°$	$254°$
SPD	$25\ (kn)$	$24.457\ (kn)$

5. IMPLEMENTATION OF A COMPUTING APPLICATION

INPUT DATA

The own ship

HDG	SPD
340.0	15.0
(degrees)	*(knots)*

The target A

		BRG	RNG
Moment t_1	0	030.0	9.00
Moment t_2	6	025.0	6.30
	(minutes)	*(degrees)*	*(nm)*

OUTPUT DATA

The target A

From the initial formulas

CPA	TCPA	CSE	SPD
1.7783926	13.049946	253.97871	24.455212
(nm)	*(minutes)*	*(degrees)*	*(knots)*

From the final formulas

CPA	TCPA	CSE	SPD
1.7783926	13.049946	253.97871	24.455212
(nm)	*(minutes)*	*(degrees)*	*(knots)*

Figure 3. Implementation of a computing application

A computing application (e.g. spreadsheet *Excel*) is applied as an example of quick determination of movement factors, with input data entered and output data received almost simultaneously.

This implementation opens another ability in application of programming language for automatic calculation.

6. COMPARISON: ANALYTIC GEOMETRY VS ARPA

There were sequences of observations of targets' movement factors during the author's shipboard training. The comparison below is between the author's method and the output data from ARPA.

Table 6. Input data

The own ship		**The target A**	*Moment t_1*	*Moment t_2*
HDG	240.6°	BRG	240.4°	237.4°
SPD	12.4 (kn)	RNG	9.63 (nm)	9.58 (nm)

Output data from the computing application applied analytic geometry:

INPUT DATA

The own ship

HDG	SPD
240.6	12.4
(degrees)	*(knots)*

The target A

		BRG	RNG
Moment t_1	0	240.4	9.63
Moment t_2	6	237.4	9.58
	(minutes)	*(degrees)*	*(nm)*

OUTPUT DATA

The target A

From the initial formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	*(minutes)*	*(degrees)*	*(knots)*

From the final formulas

CPA	TCPA	CSE	SPD
9.5545608	8.2838204	217.50316	12.775235
(nm)	*(minutes)*	*(degrees)*	*(knots)*

Figure 4. Output data

Table 7. Comparison

Value	**ARPA**	**Analytic geometry**
CPA	$9.56\ (nm)$	$9.554\ (nm)$
TCPA	$8.1\ (min)$	$8.284\ (min)$
CSE	$217.4°$	$217.503°$
SPD	$12.9\ (kn)$	$12.775\ (kn)$

7. CONCLUSIONS

Recalling the gap period ($*$) mentioned above, what if the author's method of solution on maneuvering board using analytic geometry had been thought up and developed before ARPA, i.e. before the 1960s?

As to the future, hopefully, the author's method can somehow improve the way radar/ ARPA works, for instance, in shipping.

There are some aspects, which are taken into account during the project development:

1. Calculation of **leeway angle** and **drift angle**
2. The effects from **wind** and **current**
3. Manoeuvring **prediction** and **suggestion**
4. **Collision avoidance** with many targets
5. Calculation of $\boldsymbol{BCT}$ (bow crossing time) and $\boldsymbol{BCR}$ (bow crossing range)

As to the implementation of the author's method for the future, it can be suggested that, together with AI (Artificial Intelligence), "a manoeuvring advisor" can be developed. The AI, with all input data of the own vessel and the vicinity information (i.e. meteorological condition, traffic density, etc.), can rapidly calculate and display the suggested manoeuvres to the navigator. The aftermath of the suggested manoeuvres can also be predicted and presented.

It is completely obvious that, the computer and algorithm can comply with the regulations (such as the International Regulations for Preventing Collisions at Sea 1972 - COLREGs), more precisely, with very quick suggested solutions.

It is an honour and a pride for the author that the thesis has been successful proved, as per initial stage, from the slightest idea, which suddenly appeared in the author's mind two years ago (i.e. in the spring semester, 2017), during a class period of the "Maneuvering board" course, after many observations and calculations, up to now.

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